When does a firm disclose product information?*

Frédéric Koessler † Régis Renault ‡

February 15, 2012

Abstract

A firm chooses a price and the product information it discloses to a consumer whose tastes are privately known. We provide a necessary and sufficient condition on the match function for full disclosure to be the unique equilibrium outcome whatever the costs and prior beliefs about product and consumer types. It allows for products with different qualities as well as some horizontal match heterogeneity. With independently distributed product and consumer types, full disclosure is always an equilibrium and a necessary and sufficient equilibrium condition is that all firm types earn at least the full disclosure profit.

Keywords: Consumer heterogeneity; information certification; persuasion game; unraveling of information.

JEL Classification: C72; D82; L15.

*We thank Justin Johnson, Kathryn Spier, and two anonymous referees for useful comments and suggestions. We also thank seminar participants at University of Alberta, Bonn (Max Planck Institute), CREST X-LEI, University College Dublin, University College London, HEC Paris, University of Mannheim, Université de Le Mans, University of Montreal, University of Rome Tor Vergata, University of California San Diego, University of Southern California, University of Vienna, University of Virginia, the 2010 EARIE conference, the 2010 CESifo conference on applied microeconomics, the 2nd workshop on the economics of advertising and marketing, the 3rd Transatlantic Theory Workshop, and the workshop on information transmission and persuasion in games. The first author thanks CEPREMAP and the second author thanks Institut Universitaire de France for financial support.

†Paris School of Economics – CNRS. koessler@pse.ens.fr
‡Université de Cergy-Pontoise, Thema. regis.renault@u-cregy.fr
1 Introduction

The market outcome depends to a large extent on the availability of product information to consumers before they purchase. Much attention has been devoted to the transmission of quality information from firms to consumers. Product quality is however a very special kind of product information because all consumers agree that higher quality products are better. Much of the relevant product information pertains to characteristics that appeal differently to different types of consumers and thus concerns the horizontal match between the buyer and the product. For instance, video game users usually prefer higher quality graphics, but most casual users will not find such a higher quality so appealing if it is associated with high required skills to play the game.

A firm may provide and certify product information in many different manners. A software or video game company can provide free trial on-line versions, or include a review from a third-party (such as a specialized magazine) in advertisements, on the package of the product or on the company’s website. Food and drink producers might offer free samples or display verifiable nutritive attributes. Certain product characteristics like the sound level or the energy consumption of household appliances can be tested by independent laboratories and sellers may then display the test results. More generally, laws on misleading advertising, to the extent that they are effectively implemented, enable firms to make credible claims in ads about any verifiable information pertaining to their product. This article analyzes the disclosure of certified information, allowing for horizontal match information, and investigates under what circumstances the firm will voluntarily and fully disclose product information.

We consider a monopoly seller and a buyer with unit demand. The match function indicates the consumer’s valuation as a function of his own type (his privately known taste) and the type of the firm (the product characteristics). A special case is the standard “persuasion game” (Grossman, 1981, Grossman and Hart, 1980, Milgrom, 1981, Milgrom and Roberts, 1986) where all consumer types have an identical ranking of valuations for the dif-
ferent product types. In other words, product types may be ranked in terms of quality (see Milgrom, 2008, for a recent literature review). As is well known, if the firm may perfectly certify product quality at no cost, full quality disclosure is the unique equilibrium outcome. The argument runs as follows. The top quality product type would never pool with any other product type because it can certify that it is the highest quality and sell with the same probability at a higher price. The argument unravels down to the lowest quality type.

We generalize the unraveling result by providing a necessary and sufficient condition on the match function for full revelation of the product’s type to be the unique equilibrium, regardless of the firm’s costs and the prior on the agents’ types. We call this condition pairwise monotonicity. It requires that, for every pair of types of the firm and every pair of types of the consumer, the matches can be ordered with respect to the firm’s types or with respect to the consumer’s types. Equivalently, for each pair of product types, consumer types can be partitioned into subgroups such that (i) for any two subgroups all consumer types in one subgroup are willing to pay more for the two products than all consumer types in the other subgroup, and (ii) all consumer types within a subgroup rank the two product types identically.

The standard persuasion game is clearly a special case where pairwise monotonicity trivially holds. But pairwise monotonicity can also accommodate products differentiated in terms of their horizontal match with different consumer types. For example, it holds whenever consumer types can be ranked in terms of willingness to pay in the same manner for all product types. As a more elaborate illustration, consider a firm selling a video game that may be one of three possible product types, $A$, $B$ and $C$. The situation depicted in Figure 1 is consistent with pairwise monotonicity. All gamers value $C$ less than $A$ and $B$. Furthermore, all types of hard-core gamers are willing to pay more than casual gamers for $A$ and $B$, whereas hard-core (respectively casual) gamers order $A$ and $B$ in the same way.

Pairwise monotonicity is the weakest possible sufficient condition for uniqueness of the fully revealing equilibrium outcome in the sense that, if it does not hold, there are some
prior beliefs and costs such that an equilibrium exists where information is not fully revealed. However, if the prior is restricted so that product types and consumer types are independently distributed, then full disclosure of the product’s type is always an equilibrium, even when pairwise monotonicity fails. But other (partially and non) revealing equilibria may exist. We show that when types are independent a necessary and sufficient condition for an outcome to be supported by an equilibrium is that it yields a profit at least as large as full revelation for all product types.

Finally, we explore the implications of our results in terms of the full information demands for the various product types. Interestingly, pairwise monotonicity puts no restriction on the shape of these demand curves: it holds as long as consumer types are ranked in the same manner along all the curves. We finally show that, if firm and consumer types are independently distributed, pairwise monotonicity is equivalent to the following requirement: for any two product types, the set of consumer types with willingness to pay between two crossing points of the demand curves should be the same for both product types and all
consumer types in that set rank the two products identically. This condition ensures that for any purchase probability, the inverse demand for one product type lies above the inverse demand if the two types pool. This allows for a deviation from a pooling equilibrium that is profitable for one of the two types of the firm.

*Related Literature.* Recently, Anderson and Renault (2006, 2009), Johnson and Myatt (2006) and Sun (2011) have analyzed disclosure of horizontal match information in specific examples and find that it may be profit maximizing for the firm to disclose no information or partial information. Anderson and Renault (2006) and Johnson and Myatt (2006) assume that all product types are symmetric, so that the profit maximizing solution is not type dependent. Johnson and Myatt (2006) consider a set of signals about the consumer’s valuation that may be ranked unambiguously in terms of informativeness. They find that the monopolist’s profit is maximized by using either the least informative or the most informative signal. By allowing for general informative signals, Anderson and Renault (2006) show that it is optimal for the firm to provide partial information that takes the form of a threshold on the consumer’s valuation, where those consumers with valuations above the threshold learn this information but no more.¹ Sun (2011) considers a monopolist selling a product on the Hotelling line and finds that there are equilibria where product types that are close enough to the end points pool on not disclosing product information.²

We present the model in Section 2 and then turn in Section 3 to establishing that pairwise monotonicity is sufficient for existence and uniqueness of a fully revealing equilibrium. Section 4 provides a general condition for existence of a fully revealing equilibrium and explores

---

¹Although Anderson and Renault derive the result while allowing the consumer to acquire information through search, the result still holds if search is ruled out (see Saak, 2006).

²Other explanations in the literature for partial information disclosure include competition (Board, 2009, Levin, Peck, and Ye, 2009), unsophisticated buyers (Hirshleifer, Lim, and Teoh, 2004, Mullainathan, Schwartzstein, and Shleifer, 2008), costly communication (Jovanovic, 1982, Verrecchia, 1983) and partial certifiability (Shin, 1994). Recently, Chakraborty and Harbaugh (2011) also study informative advertising in a cheap talk framework, assuming that firms can only provide “soft information” to the consumers. Other articles study optimal disclosure, assuming that the seller can commit to a disclosure rule (see, e.g., Ottaviani and Prat, 2001, Rayo and Segal, 2010, Kamenica and Gentzkow, 2011); by contrast, we assume that the seller chooses which information to disclose at the interim stage.
the possibility that other equilibria also exist. Some discussion of our results is presented in Sections 5 and 6 by interpreting our analysis in terms of demand curves and then discussing some possible extensions. Section 7 concludes.

2 Model

A monopolist sells a single unit of its product to a consumer. The firm has perfect and private information about the product’s characteristics whereas the consumer has perfect and private information about his tastes. The match between the characteristic of the firm’s product and the consumer’s tastes may therefore be written as

\[ r(s, t) \in \mathbb{R}_+ , \]

where \( s \in S \) is the firm’s type and \( t \in T \) is the consumer’s type. A consumer’s type (respectively a firm’s type) describes his private information about his tastes (respectively its private information about the product’s characteristics) as well as any information he may have about the other party’s information. For technical simplicity the sets \( S \) and \( T \) are assumed to be finite. Let \( \mu \in \Delta(S \times T) \) be the strictly positive prior probability distribution over the profile of types. The firm’s type and the consumer’s type are allowed to be correlated. This could for instance reflect a difference in information across consumer types about the product’s characteristics or difference in information across firm types about the consumer’s tastes.

The firm has a constant marginal cost \( \gamma(s) \geq 0 \) when its type is \( s \). We assume that, for every \( s \in S \), there exists \( t \in T \) such that \( r(s, t) > \gamma(s) \), meaning that each type of the firm can potentially make a strictly positive profit with at least one type of the consumer.

The timing of the game is as follows.

(i) Information stage. The firm learns \( s \in S \) and the consumer learns \( t \in T \);
(ii) **Pricing and disclosure stage.** The firm commits to an observable price \( p \in \mathbb{R} \) and sends a message \( m \in M(s) \), where \( M(s) \) is a nonempty type dependent set;

(iii) **Decision stage.** The consumer observes the price \( p \) and the message \( m \), and chooses whether to buy the good or not;

(iv) **Payoffs.** Players’ payoffs are zero when the consumer does not buy the good; if the consumer buys the good his payoff is \( r(s, t) - p \), and the firm’s payoff is \( p - \gamma(s) \).

Denote by \( M = \bigcup_{s \in S} M(s) \) the set of all possible messages that can be sent in the disclosure stage. Every subset of types of the firm is certifiable in the sense that for every \( \overline{S} \subseteq S \), there is a message \( m_{\overline{S}} \in M \) such that \( m_{\overline{S}} \in M(s) \) if and only if \( s \in \overline{S} \). Notice that under this assumption, any information that could be signaled through prices can also be transmitted at no cost with direct information disclosure. Hence, although the price may depend on the firm’s type, it has no real signaling role along the equilibrium path.

For the most part, the analysis only requires that full disclosure of the firm’s type \( s \) (message \( m_s \)) is possible. The assumption that disclosed information is certifiable is especially justified for advertising because of laws on misleading advertising, but is also justified if the firm can provide hard evidence about the product characteristic, or if disclosed information can be verified by the consumer at no cost.

A (pure) strategy of the firm is a mapping \( \varphi_F : S \rightarrow \mathbb{R} \times M \) such that \( \varphi_F(s) \in \mathbb{R} \times M(s) \) for every \( s \in S \). A (pure) strategy of the consumer is a mapping \( \varphi_C : T \times \mathbb{R} \times M \rightarrow \{\text{Buy}, \text{NotBuy}\} \). A belief function of the consumer is a mapping \( \beta : T \times \mathbb{R} \times M \rightarrow \Delta(S) \). As a solution concept for this game, we use the sequential equilibrium. The additional restrictions imposed on a sequential equilibrium, on top of those on a perfect Bayesian equilibrium, are especially relevant in this context, where beliefs are defined for a receiver (the consumer) who may have different types (see, for instance, Footnote 11).3 Our results

---

3Strictly speaking, as the set of possible signals is infinite (any price \( p \in \mathbb{R} \) is possible), a strictly positive perturbed strategy cannot be defined as in Kreps and Wilson (1982) (they consider finite games). However, it is easy to avoid this problem by assuming that the set of possible prices is finite but fine enough (all our results and examples apply with a fine enough set of prices).
focus on the existence of particular pure strategy equilibria, but our uniqueness results are proved in the class of all equilibria (pure and mixed).

After the pricing and disclosure stage, the optimal decision of the consumer of type $t$ is to buy the product if and only if

$$E[r(s, t) \mid t = t, (p, m)] \geq p.$$ 

In equilibrium, a consumer necessarily buys when indifferent.\(^5\) Hence, the optimal choice $(p^*, m^*)$ of the firm of type $s$ should satisfy

$$(p^*, m^*) \in \arg \max_{(p, m) \in \mathbb{R} \times M(s)} (p - \gamma(s))D(p, m, s),$$

where

$$D(p, m, s) \equiv \Pr [E[r(s, t) \mid t, (p, m)] \geq p \mid s = s],$$

is the expected demand of the firm when its type is $s$ and it sends the signal $(p, m)$. Notice that $E[r(s, t) \mid t, (p, m)]$ only depends on the realization of $s$ through $t$, and only if types are not independent.

Throughout the article, when we characterize fully revealing equilibria, we resort to the idea of a worst case type to conveniently characterize the consumer’s belief off the equilibrium path. Formally, we say that $s$ is a worst case type for the ordered pair of price and message $(p, m)$ if for every type $s'$ of the firm that is able to send the message $m$, the expected demand for type $s'$ when the consumer believes that the firm’s type is $s$ is not higher than when the consumer believes that the firm’s type is $s'$; the set of such worst case types for $(p, m)$ is

\(^4\) Bold letters denote random variables when there may be a risk of confusion. Expectations and probabilities are defined w.r.t. the consumer’s belief $\beta$.

\(^5\) Otherwise, the firm would brake the indifference by slightly lowering its price.
denoted by (this set might be empty):\(^6\)

\[
wct(p, m) = \{s \in M^{-1}(m) : D(p, m_s, s') \leq D(p, m_{s'}, s') \forall s' \in M^{-1}(m)\}
\]

3 Generalized unraveling

We now provide a general condition for a fully revealing equilibrium to exist and be unique whatever the costs and the prior. We say that the match is \textit{statewise monotonic with respect to the firm’s type}, if, for every \(s, s' \in S, s \neq s'\), either \(r(s', t) > r(s, t)\) for all \(t \in T\) or \(r(s', t) < r(s, t)\) for all \(t \in T\). There is then a quality ranking of product types as in the standard persuasion game or the following example:

\[
\begin{array}{ccc}
&t_1&t_2&t_3 \\
s_1&9&5&7 \\
s_2&8&3&6 \\
s_3&6&2&0 \\
\end{array}
\]

It is easy to show that existence and uniqueness of the fully revealing equilibrium is guaranteed in this case. Existence is obtained by considering beliefs that put probability one on the following price independent worst case type:

\[
wct(p, m) = \arg \min_{s \in M^{-1}(m)} r(s, t),
\]

for some \(t \in T\). Thanks to statewise monotonicity with respect to the firm’s type, the above worst case type is independent of the selected consumer type \(t\) and pins down the “lowest quality” product.

\(^6\)Notice that if consumer types always assign probability one to a worst case type independently of their own type, then it can readily be established that the consistency requirement in the definition of a sequential equilibrium is satisfied whatever the (strictly positive) prior.
Deviation from full disclosure, that involves selecting \( m \neq m_s \) is not profitable because 
\[ D(p, m, s) \leq D(p, m_s, s) \]
for all \( m \in M(s) \) and \( p \in \mathbb{R}_+ \). To show that there is no other equilibrium outcome assume by way of contradiction that types in a subset with at least two elements \( \overline{S} \subseteq S \), send the same signal \((p, m)\) with strictly positive probability. Let

\[ \bar{s} = \arg \max_{s \in \overline{S}} r(s, t), \]

for any \( t \in T \), be the highest quality product in \( \overline{S} \). If \( \bar{s}'s \) demand is zero, then by certifying its type it can apply a price higher than \( \gamma(\bar{s}) \) and earn a strictly positive profit. If \( \bar{s}'s \) demand is strictly positive, then by statewise monotonicity with respect to the firm’s type the willingness to pay with full revelation of type \( \bar{s} \), \( r(\bar{s}, t) \) exceeds the willingness to pay if \( \bar{s} \) pools, for all \( t \in T \), so the type \( \bar{s} \) firm can increase its price when it certifies its type, while keeping the same demand as when it pools with types in \( \overline{S} \). Hence pooling can never be sustained in equilibrium.

We next show by means of examples that statewise monotonicity with respect to product types is much too strong a condition for existence and uniqueness of a fully revealing equilibrium. Consider first the following match:

\[
\begin{array}{c|cc}
  & t_1 & t_2 \\
\hline
s_1 & 1 & 6 \\
s_2 & 2 & 3 \\
\end{array}
\]

This match function is not statewise monotonic with respect to the firm’s type, but it is statewise monotonic with respect to the consumer’s type in the sense that for every \( t, t' \in T, t \neq t' \), either \( r(s, t') > r(s, t) \) for all \( s \in S \) or \( r(s, t') < r(s, t) \) for all \( s \in S \).\(^7\) It is easy to see

\(^7\)The following application from Chakraborty and Harbaugh (2010) also satisfies statewise monotonicity with respect to the consumer’s type but not with the respect of the firm’s type: the firm’s characteristic is multidimensional, \( s = (s^1, s^2) \in \mathbb{R}_+^2 \), where \( s^1 \) is an horizontal aspect of the product, \( s^2 \) a vertical aspect, the firm is located at 0, and the valuation of a consumer located at \( t \in [0,1] \) is \( r(s^1, s^2, t) = s^2 - s^1 t \). In Chakraborty and Harbaugh (2010) full disclosure is not an equilibrium because they consider the case of cheap talk (i.e., information is not certifiable).
that whatever the prior the unique equilibrium is fully revealing. To get existence it suffices to consider the following worst case type for \( m \in M(s_1) \cup M(s_2) \):

\[
\text{wct}(p, m) = \begin{cases} 
  s_2 & \text{if } p > 2, \\
  s_1 & \text{if } p \leq 2.
\end{cases}
\]

In other words, the worst case type is the product type that the marginal consumer type likes the least. To get uniqueness, suppose on the contrary that \( s_1 \) and \( s_2 \) pool. Then the firm type selling the product that suits best the marginal consumer will deviate. If both \( t_1 \) and \( t_2 \) buy, then the price is given by \( p \in (1, 2) \) and type \( s_2 \) can profitably deviate by revealing its type and charging price \( p' = 2 > p \), in which case both types of the consumer still buy. Similarly, if only \( t_2 \) buys, then the price is given by \( p \in (3, 6) \) and type \( s_1 \) can profitably deviate by revealing its type and charging price \( p' = 6 > p \), in which case consumer \( t_2 \) still buys.

Consider now the following match function:

\[
\begin{array}{ccc}
  t_1 & t_2 & t_3 \\
  s_1 & 5 & 4 & 3 \\
  s_2 & 6 & 1 & 2 \\
\end{array}
\]

This match function is neither statewise monotonic with respect to the firm’s type nor statewise monotonic with respect to the consumer’s type. Nevertheless, the unique equilibrium is fully revealing whatever the prior. To see that there is a fully revealing equilibrium it suffices to consider the following worst case type for \( m \in M(s_1) \cup M(s_2) \):

\[
\text{wct}(p, m) = \begin{cases} 
  s_2 & \text{if } p \leq 4, \\
  s_1 & \text{if } p > 4.
\end{cases}
\]

Again, it is the worst product for the marginal consumer type: for \( p \leq 4 \), type \( t_1 \) cannot
be marginal because the firm would otherwise charge a price of at least 5. To see that this
equilibrium outcome is unique, suppose on the contrary that \( s_1 \) and \( s_2 \) pool. If \( t_3 \) is the
marginal type, then to prevent \( s_1 \) from revealing its type the price should not be smaller
than 3, a contradiction with the fact \( t_3 \) buys. If \( t_2 \) is the marginal consumer, then to prevent
\( s_1 \) from revealing its type the price should not be smaller than 4, a contradiction with the
fact \( t_2 \) buys. Finally if only \( t_1 \) buys, then \( s_2 \) would deviate by revealing its type and choosing
a price equal to 6.

The three match functions that we have considered thus far share a common feature that
turns out to be a key property for establishing existence and uniqueness of a fully revealing
equilibrium. It runs as follows. For any non empty subset of firm types, \( S \subseteq S \), and any non
empty subset of consumer types, \( T \subseteq T \), there exists an ordered pair \((\bar{s}, \bar{t}) \in S \times T \) that has
the following “saddle point” property: (i) \( r(\bar{s}, \bar{t}) = \max_{s \in S} r(s, \bar{t}) \); (ii) \( r(\bar{s}, \bar{t}) = \min_{t \in T} r(\bar{s}, t) \).
In other words, product type \( \bar{s} \) is the best product in \( S \) for consumer type \( \bar{t} \) and consumer
type \( \bar{t} \) is willing to pay the least among all consumer types in \( T \) for product type \( \bar{s} \). For
instance, such a saddle point is reached for the whole \( 3 \times 3 \) matrix in the match \( r_A \), which
satisfies statewise monotonicity with respect to the firm’s type, at \( (\bar{s}, \bar{t}) = (s_1, t_2) \). Similarly,
the \( 2 \times 2 \) matrix has a saddle point at \( (s_2, t_1) \) in the match \( r_B \), which that satisfies statewise
monotonicity with respect to consumer types, and at \( (s_1, t_3) \) in our last \( 2 \times 3 \) example, \( r_C \).
If we drop \( t_3 \) in that same last example then the saddle point is reached at \( (s_1, t_2) \).

We now show that if the above property holds, then there cannot be an equilibrium with
several firm types pooling with some positive probability, as long as the match function is
generic.\(^8\) Consider some pooling of several firm types in \( S \), where the price is such that the
set of consumer types who buy is \( T \). Then the price is less than consumer \( \bar{t} \)’s expectation of
the match, which, if the match function is generic, is strictly less than \( r(\bar{s}, \bar{t}) \). Hence, firm

\(^8\)Formally, we only require that \( r(s, t) \neq r(s', t) \) for every \( s \neq s' \) and \( t \in T \). This assumption rules out
situations where two firm types correspond to the same product and differ only in terms of the available
information about the other party. If we allow for different firm types selling the same product, so that the
match would be constant across those firm types, then it is possible to adapt the argument to show that
product information is revealed in equilibrium. This is the case if product information is certifiable and the
consumer is never indifferent between two different products.
type $s$ could increase its price to $r(s, t)$ and still sell to all types in $T$ by part (ii) of the saddle point property above.

If we consider the restriction of the match to two firm types and two consumer types, then the saddle point property is equivalent to statewise monotonicity with respect to the firm’s type or the product’s type. This property holds for all pairs of product types and consumer types in all three match functions above. We now state this property formally.

**Definition 1** The match function $r(\cdot, \cdot)$ is **pairwise monotonic** if for every pair of types of the firm $(s, s') \in S^2$, $s \neq s'$, and every pair of types of the consumer $(t, t') \in T^2$, $t \neq t'$, one of the following conditions hold:

- **(i)**
  \[
  \begin{cases}
  r(s, t) > r(s', t) \\
  r(s, t') > r(s', t');
  \end{cases}
  \]

- **(ii)**
  \[
  \begin{cases}
  r(s', t') > r(s', t) \\
  r(s', t') > r(s, t');
  \end{cases}
  \]

- **(iii)**
  \[
  \begin{cases}
  r(s, t) > r(s, t') \\
  r(s', t) > r(s', t');
  \end{cases}
  \]

- **(iv)**
  \[
  \begin{cases}
  r(s, t') > r(s, t) \\
  r(s', t') > r(s', t).
  \end{cases}
  \]

The proof of the next theorem establishes that pairwise monotonicity is equivalent to the existence of a saddle point in all sub-matrices of the match function. From our discussion above, this establishes that there cannot be an equilibrium that is not fully revealing.

**Remark 1** The equivalence of pairwise monotonicity and the existence of a saddle point for all sub-matrices of the match function has an interesting alternative interpretation. It says that if a zero-sum game is pairwise monotonic, then every sub-matrix has a saddle point. Hence, every sub-matrix, interpreted as a zero-sum game, is strictly determined in the sense that the pure maxmin coincides with the pure minmax. Coincidentally, this is related to Shapley (1964) who shows that a zero-sum game has a (pure) saddle point if every $2 \times 2$ sub-matrix of the game has a saddle point.
Theorem 1 If the match is generic and pairwise monotonic, then there is a unique sequential equilibrium outcome, which is fully revealing.

Proof. Uniqueness. Assume that there exists an equilibrium where a set of types $S \subseteq S$, with $|S| \geq 2$, pool (i.e., choose the same signal $(p, m)$ with strictly positive probability). Let $T \subseteq T$ be the set of consumer’s types that buy the good after the signal $(p, m)$. For every $s \in S$ there exists $t \in T$ such that $r(s, t) > \gamma(s)$, so we necessarily have $T \neq \emptyset$ and $p > \gamma(s)$ for all $s \in S$. When $|T| = 1$, the standard unraveling argument shows that pooling is impossible, so let $|T| \geq 2$. Denote by $R = (r(s, t))_{(s, t) \in S \times T}$ the matrix of matches restricted to types in $S \times T$. Notice that the price applied by firms in $S$ is such that

$$p \leq \min_{t \in T} E(r(s, t) \mid (p, m)).$$

For every $s \in S$ and $t \in T$ let

$$l(s) \in \arg\min_{t \in T} r(s, t) \quad \text{and} \quad \bar{s}(t) = \arg\max_{s \in S} r(s, t).$$

That is, $l(s)$ is (possibly a selection of) the smallest match in the $s$ line of $R$ (call it a white cell) and $\bar{s}(t)$ is the highest match in the $t$ column of $R$ (call it a gray cell). Equilibrium conditions imply that those (gray and white) cells cannot by confounded; that is, there is no ordered pair $(s, t)$ such that $(s, t) = (\bar{s}(t), l(s))$. Otherwise, firm $s$ can profitably deviate by revealing its type and applying the price $r(s, t) = r(\bar{s}(t), t) = \max_{s' \in S} r(s', t) > p$, for which all consumers in $T$ buy as $r(s, t) = r(s, l(s)) \leq r(s, t')$ for every $t' \in T$.

Now, in the matrix $R$ we delete iteratively any line without a gray cell, i.e., any line $s$ such that $s \neq \bar{s}(t)$ for every $t \in T$, and any column without a white cell, i.e., any column $t$ such that $t \neq l(s)$ for every $s \in S$. Denote by $R^*$ the remaining matrix of matches, and $S^*$ and $T^*$ the corresponding sets of types of the firm and of the consumer. Notice that this matrix has at least two lines and two columns because gray and white cells cannot be
confounded. This procedure is illustrated below.

\[
\overline{R} = \begin{pmatrix}
\begin{array}{cccc}
\blacksquare & \blacksquare & \cdot & \cdot \\
\cdot & \blacksquare & \blacksquare & \cdot \\
\cdot & \cdot & \blacksquare & \blacksquare \\
\cdot & \cdot & \cdot & \blacksquare \\
\end{array}
\end{pmatrix} \rightarrow \begin{pmatrix}
\begin{array}{cccc}
\blacksquare & \cdot & \cdot & \cdot \\
\cdot & \blacksquare & \cdot & \cdot \\
\cdot & \cdot & \blacksquare & \cdot \\
\cdot & \cdot & \cdot & \blacksquare \\
\end{array}
\end{pmatrix} \rightarrow \begin{pmatrix}
\begin{array}{cccc}
\blacksquare & \blacksquare & \cdot & \cdot \\
\cdot & \blacksquare & \cdot & \cdot \\
\cdot & \cdot & \blacksquare & \blacksquare \\
\cdot & \cdot & \cdot & \blacksquare \\
\end{array}
\end{pmatrix} = R^*
\]

Let

\[r^* = \max_{(s,t) \in S^* \times T^*} r(s, t),\]

be the highest match in the matrix $R^*$. By definition $r^*$ necessarily corresponds to a gray cell, i.e., $r^* = r(s^*(t^*), t^*)$ for some $t^* \in T^*$. Let $s^* = \bar{s}(t^*)$ so that $r^* = r(s^*, t^*)$. By the construction of $R^*$ (any column having a white cell and any line having a gray cell) there exists $s' \in S^*$ such that $\ell(s') = t^*$ and $t' \in T^*$ such that $\bar{s}(t') = s'$ as illustrated below:

\[
\begin{pmatrix}
\begin{array}{cccc}
\blacksquare & \blacksquare & \cdot & \cdot \\
\cdot & \blacksquare & \cdot & \cdot \\
\cdot & \cdot & \blacksquare & \cdot \\
\cdot & \cdot & \cdot & \blacksquare \\
\end{array}
\end{pmatrix} = \begin{pmatrix}
\begin{array}{cccc}
\blacksquare & \blacksquare & \cdot & \cdot \\
\cdot & \blacksquare & \cdot & \cdot \\
\cdot & \cdot & \blacksquare & \cdot \\
\cdot & \cdot & \cdot & \blacksquare \\
\end{array}
\end{pmatrix} = R^*
\]

Finally, $r(s^*, t') < r(s^*, t^*) \equiv \max_{(s,t) \in S^* \times T^*} r(s, t)$, so the match is not pairwise monotonic.

Existence. Consider a complete disclosure strategy of the firm such that a message $m_s \in M(s)$ is sent by each type of the firm, with $m_s \notin M(s')$ for all $s' \neq s$. We construct a worst case type function such that no firm’s type has an incentive to deviate from this complete disclosure strategy. Consider a signal $(p, m)$ off the equilibrium path, and consider the matrix of matches restricted to firm types in $M^{-1}(m)$, with $|M^{-1}(m)| \geq 2$. By the first part of the proof we know that if the match is pairwise monotonic, then the minimum match of some firm’s type coincides with the maximum match of some consumer’s type (for every sub-matrix of matches); that is, at least one white cell coincides with some gray cell. Denote by $(s_1, t_1)$ the corresponding cell, and $r_1 = r(s_1, t_1)$ the corresponding value. If $p > r_1$ then
type $t_1$ never buys whatever his beliefs, and we apply the same construction to the matrix of matches without $t_1$. If $p \leq r_1$ then type $s_1$ has no incentive to deviate from full disclosure to $(p, m)$ and we let $wct(p, m) \neq s_1$ (i.e., we consider a belief system such that $\beta(s_1 | t, m, p) = 0$ for every $t$). If $|M^{-1}(m)\{s_1\}| = 1$ the argument is complete. Otherwise we consider the new matrix of matches without the $s_1$ line. We apply the same reasoning as above to this new matrix: let $(s_2, t_2)$ be a gray and white cell of this matrix, let $r_2 = r(s_2, t_2)$ be the corresponding value, and construct the worst case types as above. If $p > r_2$ then type $t_2$ never buys whatever his beliefs, and we apply the same construction to the matrix of matches without $t_2$. If $p \leq r_2$ then type $s_2$ has no incentive to deviate from full disclosure to $(p, m)$ and we let $wct(p, m) \neq s_2$. If $|M^{-1}(m)\{s_1, s_2\}| = 1$ the argument is complete. Otherwise, we apply the same construction up to $(s_k, t_k)$ such that $|M^{-1}(m)\{s_1, s_2, \ldots, s_k\}| = 1$. ■

As will be seen in Proposition 3 below, pairwise monotonicity is the weakest possible sufficient condition on the match function that ensures uniqueness, while keeping prior beliefs and costs unrestricted.

For the sake of interpretation, we now provide the following alternative definition of pairwise monotonicity, which is useful to relate the properties of match functions to inverse demand functions (see Section 5). A match function is pairwise monotonic if for every pair of product types, consumer types can be partitioned into subgroups such that (i) there is monotonicity with respect to consumer types between subgroups (ii) all consumers in a subgroup rank the two product types identically.\(^9\) For instance, in the video game example in the introduction, there are two groups of consumer types, the casual users and the hard-core users. The ranking between the two high quality products is the same within subgroups, although it differs across subgroups. An illustration with four subgroups is depicted in Figure 2, assuming for clarity a continuum of consumer types: consumer types are on the horizontal axis whereas the vertical axis measures the match. There is no loss of generality in taking the match for $s_1$ to be increasing because consumer types may always be rearranged.

\(^9\)Of course, this interpretation also works by switching “product” and “consumer” types.
Pairwise monotonicity, which is satisfied in Figure 2 (a), is equivalent to requiring that, for any crossing points between the two matches, the match with product \( s_2 \) lies below the crossing point for consumer types on the left and above the crossing point for consumer types to the right. For a pair of consumer types between two crossing points, statewise monotonicity with respect to product types holds, whereas for a pair of consumer types on either side of a crossing point, statewise monotonicity with respect to consumer types holds. If the match for product \( s_2 \) loops up before a crossing point or loops down after a crossing point, then pairwise monotonicity fails as in Figure 2 (b).

We next consider some equilibrium properties when pairwise monotonicity does not necessarily hold.

### 4 Equilibria without unraveling

The next proposition shows that, even if pairwise monotonicity fails, there still exists a fully revealing equilibrium, whatever the prior as long as there is no correlation between the firm’s and the consumer’s type. To understand why, notice that if the type of the firm is
independent of the type of the consumer, then the demand of the firm given \( p \) and \( m \) does not depend on its actual type, i.e.,

\[
D(p, m, s) = D(p, m) = \Pr[E[r(s, t) | t, (p, m)] \geq p].
\]

It follows that the firm’s preference over the consumer’s belief is, conditionally on the price and the message sent, independent of its real type because the demand only depends on the price and the consumer’s belief. Hence, starting from a strategy of full disclosure it is easy to prevent the firm from deviating by constructing, for each price \( p \), a worst case type that minimizes the firm’s demand at price \( p \).\(^{10}\) This can be done independently of the firm’s real type.

**Proposition 1** Assume that the consumer’s type and the firm’s type are independent. Then, there exists a fully revealing equilibrium.

*Proof.* Consider a complete disclosure strategy of the firm such that a message \( m_s \in M(s) \) is sent by each type of the firm, with \( m_s \notin M(s') \) for all \( s' \neq s \). When types are independent, the demand for such a message at price \( p \) is given by

\[
D(p, m_s) = \Pr[r(s, t) \geq p].
\]

For every \( m \in M \) and \( p \in \mathbb{R} \), let

\[
wct(p, m) \in \arg\min_{s \in M^{-1}(m)} D(p, m_s),
\]

be the (worst case) type of the firm whose demand at price \( p \) when it reveals its type to the consumer is the lowest one among all types that can send message \( m \). For any signal \((p, m) \in \mathbb{R} \times M\) of the firm, consider the belief of the consumer that puts probability one on \(^{10}\)When the cost of the firm is not type dependent, the proof of Proposition 1 can be simplified by considering price-independent beliefs off the equilibrium path.
Along the equilibrium path, the firm gets
\[
\max_{p_s}(p_s - \gamma(s))D(p_s, m_s).
\]
This profit is larger than what it gets by deviating to \((p, m)\in \mathbb{R} \times M(s)\), which by construction is equal to
\[
(p - \gamma(s)) \min_{s'\in M^{-1}(m)} D(p, m_{s'}).\]
This completes the proof of Proposition 1. ■

The appendix describes an example with correlated types (but without pairwise monotonicity) where there is no fully revealing equilibrium.\(^{11}\) Proposition 1 shows that as long as types are independent, pairwise monotonicity is not necessary for existence of a fully revealing equilibrium. As we show below, pairwise monotonicity is however necessary to ensure uniqueness of the fully revealing outcome whatever the prior distribution of types and costs. We start by providing a useful characterization of equilibria with independent types.

The next proposition provides a simple method for characterizing all equilibrium outcomes when types are independent. To this end we define canonical disclosure strategies: a strategy \(\varphi_F(\cdot)\) is canonical if, for every price and message \((p, m)\) in the range of \(\varphi_F(\cdot)\), if type \(s\in S\) does not send \((p, m)\) (i.e., \(\varphi_F(s) \neq (p, m)\)), then the message \(m\) cannot be sent by type \(s\) (i.e., \(m \notin M(s)\)). Notice that, under our assumption that every subset of types is certifiable, considering only canonical strategies does not restrict the set of possible equilibrium outcomes.\(^{12}\) The proposition says that a (canonical) disclosure strategy induces an equilibrium outcome of the game if and only if the (interim) payoff for the firm is no smaller than the payoff the firm would earn at the fully revealing equilibrium whatever its

---

\(^{11}\)In the example, it is important to require strong belief consistency (in the sense of Kreps and Wilson, 1982). If we allow arbitrary beliefs off the equilibrium path, existence of a fully revealing equilibrium is immediate even with correlated types. It suffices to consider beliefs off the equilibrium path that put probability one on \(\arg\min_{s\in M^{-1}(m)} r(s, t)\), which are inconsistent when they depend on \(t\).

\(^{12}\)For example, if \(S = \{s_1, s_2\}, M(s_1) = \{m_0, m_1\}, M(s_2) = \{m_0, m_2\}\), then the disclosure strategy \(\varphi_F(s_1) = (p, m_1)\) and \(\varphi_F(s_2) = (p, m_0)\) is not canonical, but is equivalent to the canonical strategy \(\varphi'(s_1) = (p, m_1)\) and \(\varphi'(s_2) = (p, m_2)\) in terms of information that is transmitted to the consumer.
Proposition 2 Assume that the consumer’s type and the firm’s type are independent. A canonical disclosure strategy induces an equilibrium iff whatever the firm’s type the induced interim expected payoff for the firm is not smaller than its fully revealing equilibrium payoff.

Proof. Necessity. This part is obvious.\(^{14}\) If the disclosure strategy of the firm is such that \(\varphi_F(s) = (p, m)\) with

\[
(p - \gamma(s))D(p, m) < (p_s - \gamma(s))D(p_s, m_s),
\]

where \((p_s, m_s)\) is the signal sent by the firm at the fully revealing equilibrium, then (simply by subgame perfection) the firm can profitably deviate from \(\varphi_F\) at \(s\) by sending the signal \((p_s, m_s)\).

Sufficiency. Consider a canonical disclosure strategy \(\varphi_F\) of the firm such that \(\varphi_F(s) = (p, m)\) and

\[
(p - \gamma(s))D(p, m) \geq (p_s - \gamma(s))D(p_s, m_s), \tag{1}
\]

where \((p_s, m_s)\) is the signal sent by the firm of type \(s\) at the fully revealing equilibrium. Notice that any deviation from a canonical disclosure strategy is observable by the consumer. Consider the same beliefs for the consumer off the equilibrium as in the proof of Proposition 1. If the firm deviates from \(\varphi_F(s) = (p, m)\) to \((p', m')\) it gets

\[
(p' - \gamma(s)) \min_{s' \in M^{-1}(m')} D(p', m_{s'}) \leq \max_{p_s} (p_s - \gamma(s)) \min_{s' \in M^{-1}(m')} D(p_s, m_{s'}) \leq \max_{p_s} (p_s - \gamma(s))D(p_s, m_s) \text{ because } s \in M^{-1}(m'),
\]

which by (1) is smaller than the payoff it gets without deviating from \(\varphi_F(s)\). \(\blacksquare\)

\(^{13}\)The proposition does not extend to correlated types because we already observed in the example in the appendix that full revelation may not be an equilibrium when types are correlated.

\(^{14}\)Notice that this part also applies with correlated types and non-canonical disclosure strategies.
Using Proposition 2 we may now show that the converse of Theorem 1 is also true in the sense that pairwise monotonicity is the weakest sufficient condition that ensures that the fully revealing outcome is unique, independent of the specification of the prior and production costs.

**Proposition 3** Assume that the match is generic but not pairwise monotonic. Then, there exists an open set of costs and priors such that the game has an equilibrium outcome, which is not fully revealing, and which is strictly better for the firm than the fully revealing equilibrium.

**Proof.** If the match is generic but not pairwise monotonic then there exist two pairs \((s_1, s_2) \in S^2\) and \((t_1, t_2) \in T^2\), where \(r(s_1, t_1) = a, r(s_1, t_2) = b, r(s_2, t_1) = c\) and \(r(s_2, t_2) = d\) such that \(a \geq b\), \(a > c\), \(d \geq c\) and \(d > b\), as illustrated below:

\[
\begin{pmatrix}
  a & \geq & b \\
  \lor & \land \\
  c & \leq & d
\end{pmatrix}
\]

Assume zero costs. Also assume for now that the prior puts all the weight on this \(2 \times 2\) matrix. Because no consumer type has a higher willingness to pay for both firm types than the other, (here \(a \geq b\) and \(d \geq c\)), we may specify a probability for firm type \(s_1, \sigma\), such that the two consumer types have the same expected willingness to pay if the firm’s type is not revealed: \(\sigma a + (1 - \sigma) c = \sigma b + (1 - \sigma) d\) (the left-hand side is larger for \(\sigma = 1\), and the reverse inequality holds for \(\sigma = 0\)). Hence, if the firm does not disclose, it can sell with probability 1 at a price equal to the common expected match.

If the firm wishes to sell with probability 1 while revealing its type, it must charge a price which is at most the lowest match for its product, \(b\) for type \(s_1\) and \(c\) for type \(s_2\). However, these values also correspond to the lowest match for one of the consumer types (there would otherwise be a saddle point), and hence are strictly less than the expected match under no disclosure. The firm therefore earns strictly less profit than with no disclosure by revealing
its type and selling with probability 1.

Finally, if the firm reveals while only selling to one consumer type, the best it can do is sell to the consumer type with the highest match with its product type and charge the corresponding price, $a$ for type $s_1$ and $d$ for types $s_2$. Assume that type $t_1$ has probability $\sigma$ and type $t_2$ has probability $1 - \sigma$. Then the corresponding revenue with such a strategy is still strictly less than the no disclosure revenue.

Let us now alter probabilities slightly so that all firm types and consumer types other than $s_1, s_2, t_1,$ and $t_2$ have a negligible but strictly positive weight, and consider strictly positive but small enough costs. Then it is still true that types $s_1$ and $s_2$ are strictly better off pooling than they would be with full disclosure. It follows from Proposition 2 that there is an equilibrium where $s_1$ and $s_2$ pool and all the other firm types fully reveal.

We illustrate the multiplicity of equilibria when pairwise monotonicity is violated with an example where it is easy to identify the equilibrium that is most favorable to the firm. Consider the following match function, which is inspired by Table 1 in Anderson and Renault (2006), with a uniform prior and the same cost $\gamma(s) = \gamma \in [0, 5)$ for every product type.

$$
\begin{array}{ccc}
 & t_1 & t_2 & t_3 \\
 s_1 & 5 & 4 & 1 \\
 s_2 & 5 & 1 & 4 \\
 s_3 & 4 & 5 & 1 \\
 s_4 & 1 & 5 & 4 \\
 s_5 & 4 & 1 & 5 \\
 s_6 & 1 & 4 & 5 \\
\end{array}
$$

The fully revealing equilibrium leads to the profit max \( \left\{ \frac{8-2\gamma}{3}, \frac{5-\gamma}{3} \right\} \) whatever the firm’s type. No information revelation yields the profit max \( \left\{ 0, \frac{10-3\gamma}{3} \right\} \), so it is an equilibrium if and only if max \( \left\{ 0, \frac{10-3\gamma}{3} \right\} \geq \max \left\{ \frac{8-2\gamma}{3}, \frac{5-\gamma}{3} \right\} \), i.e., $\gamma \leq 2$. However, when $\gamma \in [1, 4]$ the best equi-
librium for the firm is partially revealing: the firm’s disclosure strategy yields the partition 
\[{\{s_1, s_3\}, \{s_2, s_5\}, \{s_4, s_6\}}\], the price is equal to 9/2 and the profit is equal to \(\max \{0, \frac{9}{2} - \frac{2}{3}\}\). Note that this partially revealing equilibrium implements a threshold of 4 along the lines of the optimal solution described in Anderson and Renault (2006): all consumer types with willingness to pay above 4 learn this with no additional information.\(^\text{15}\)

Also note that, for any cost \(\gamma\), the profit maximizing equilibrium among the three considered above implements a socially first-best outcome because a consumer buys if and only if his match exceeds marginal cost. This is also the profit maximizing solution because the firm extracts all of the consumer’s expected surplus through its price. Hence, as the unit cost increases, the profit maximizing solution is: first, non disclosure, then partial disclosure, and finally full disclosure.

5 Demand curves and information disclosure

As illustrated by Johnson and Myatt (2006), the strategic choice by a firm to reveal or not reveal product information amounts to choosing among different demand curves. For instance, the classic unraveling result may be loosely described as follows: high quality firms prefer to reveal product information because it puts them on a demand curve that dominates the one they would face by pooling with lower quality firms. We here explore how pairwise monotonicity relates to demand curve properties.

For each product type, \(s\), we may define the perfect information inverse demand as

\[
P(q, s) \equiv \max \{p : D(p, m_s, s) \geq q\} = \max \{p : \Pr[r(s, t) \geq p | s = s] \geq q\},
\]

for any \(q \in (0, 1]\). The inverse demand gives the highest price that the firm selling product \(s\) can charge while being able to sell with a probability of at least \(q\).

\(^\text{15}\)In Anderson and Renault (2006) where consumers may acquire full product information through costly search before buying, the coincidence of profit maximization and the first-best socially optimum outcome only arises if search costs are large enough.
A first question is whether there are properties of inverse demands for the various product types that are sufficient to guarantee that the match is pairwise monotonic. One situation where we might expect this to happen is when full information demands do not cross. Such a property holds for instance when products may be ranked in terms of quality so that the match function is statewise monotonic with respect to firm types. A generalization of the persuasion game unraveling result would be that the no crossing point property of demand curves is enough to guarantee full revelation of product information. The following example however shows that this is not the case:

\[
\begin{array}{cc}
  t_1 & t_2 \\
  s_1 & 5 & 3 \\
  s_2 & 2 & 4 \\
\end{array}
\]

Assume that the prior is uniform. The inverse demand for product type \( s_1 \) always lies above the inverse demand for product \( s_2 \): that is, the largest price at which product \( s_1 \) may be sold with a given probability is always at least as large as the largest price at which product \( s_2 \) may be sold with the same probability. The match however is not pairwise monotonic and non revelation may be sustained as an equilibrium, for instance with zero production costs. It yields a profit of 3.5 against only 3 for the largest full information profit. Non disclosure is therefore an equilibrium by Proposition 2.

The above example may be enriched to illustrate an interesting property of quality disclosure. Consider now the match function below, involving four products.

\[
\begin{array}{cc}
  t_1 & t_2 \\
  s_1 & 5 & 3 \\
  s_2 & 2 & 4 \\
  s_3 & 4 & 2 \\
  s_4 & 3 & 5 \\
\end{array}
\]
Note that product type \( s_1 \) dominates product type \( s_3 \) in terms of demand curve but also in terms of quality ranking because it is preferred by both consumer types. The same is true regarding product type \( s_4 \) vis-a-vis product type \( s_2 \). The two superior quality firm types \( s_1 \) and \( s_4 \) would like to separate themselves from the two other types. Still, assuming again a uniform prior and zero costs, non disclosure may be sustained as an equilibrium, as a direct application of Proposition 2. Hence, quality information is not necessarily revealed in equilibrium when the relevant product information also pertains to the match. Sun (2011) derives related results in a setting where the set of product types and consumer types are represented by the Hotelling line and product types may also differ in terms of quality.

In order to obtain a demand dominance condition that implies pairwise monotonicity, we need to require that for any pair of products, the lowest point on one demand curve is above the highest point on the other demand curve. Formally, for any \( s, s' \in S \), we have \( P(1, s) > P(0, s') \) or \( P(1, s') > P(0, s) \).\(^{16}\) This is a very strong restriction that actually implies statewise monotonicity with respect to the firm’s type.

Conversely, pairwise monotonicity, and hence uniqueness of a fully revealing outcome may be ensured for any set of perfect information demand curves. Recall that a special case of pairwise monotonicity is statewise monotonicity with respect to consumer types. Now consider a finite set of step inverse demand functions defined on \((0, 1]\). Then it is not too difficult to see that all consumers can be ranked identically along all demand curves and the prior can be specified appropriately so that statewise monotonicity with respect to consumer types is satisfied and the resulting inverse demands are as desired.

We illustrate this idea with the following simple example. Consider the two following inverse demand functions: \( P(q, s_1) = 3 \) for \( q \in (0, .4] \), \( P(q, s_1) = 2 \) for \( q \in (.4, 1] \), \( P(q, s_2) = 4 \) for \( q \in (0, .3] \), \( P(q, s_2) = 3 \) for \( q \in (.3, .7] \) and \( P(q, s_2) = 1 \) for \( q \in (.7, 1] \). Such inverse demands may be generated, for example, with three consumer types and the following match

\(^{16}\)The inverse demand \( P \) is not defined for \( q = 0 \) and \( P(0, s) \) should be thought of as the limit as \( q \) tends to zero, which always exists.
function

\[
\begin{array}{ccc}
  r & t_1 & t_2 & t_3 \\
  s_1 & 3 & 2 & 2 \\
  s_2 & 4 & 3 & 1
\end{array}
\]

which is statewise monotonic with respect to the consumer’s type. Then assume that the prior is such that the conditional probability of consumer types \( t_1, t_2 \) and \( t_3 \) are respectively, .4, .3 and .3, conditional on \( s = s_1 \) and .3, .4 and .3, conditional on \( s = s_2 \). Such a procedure may clearly be adapted to any finite set of step inverse demand functions so that pairwise monotonicity puts no restriction on the full information demand curves for the various product types.

We now show that pairwise monotonicity imposes some restrictions as to the allocation of different consumer types along the various demand curves. This in turn provides a simple demand curve intuition on the sufficiency of pairwise monotonicity for uniqueness of the fully revealing outcome. Let us now assume that consumer and product types are independent. To illustrate the general idea, consider the example depicted in Figure 3. The match function for two product types \( s_1 \) and \( s_2 \) with a continuum of consumer types is shown in panel (a). It satisfies pairwise monotonicity. The corresponding demand curves are derived in panel (b), assuming that consumer types are uniformly distributed.

Because of independence and pairwise monotonicity, crossing points for the match functions in Figure 3 (a) exactly translate into crossing points between inverse demand curves in Figure 3 (b). Take for instance consumer types to the left of the first crossing point between matches. They are all willing to pay less than the match value at the crossing point and all other consumer types are willing to pay more. Hence the probability of selling at that price is given by the probability measure associated to these types which is the same for both products, thanks to our independence assumption. Hence demand curves must cross at this price and this corresponds to the farthest right crossing point in Figure 3 (b). Repeating the argument iteratively shows that all crossing points coincide. From this analysis we conclude
that the consumer type population may be partitioned into subgroups such that the willingness to pay for both products within a given subgroup lies between two consecutive crossing points of the demand curves and all consumers within a subgroup agree on the ranking of the two products.

Figure 3 also shows as a dashed curve the expected match $E[r(s, t)]$ for all consumer types in Panel (a) (assuming product types have equal probabilities) from which is derived the no information inverse demand in Panel (b), again as a dashed curve. The expected demand lies strictly in between the two demand curves between two crossing points as long as the probability defined on product types is non-degenerate. Hence, if we consider an equilibrium where the two products pool and face the no information demand curve, there is always one firm type that is better off deviating to full information thus selling at a higher price with the same probability.\footnote{This argument works as long as the price in the candidate pooling equilibrium does not coincide with a crossing point. If it does, then a standard argument using first order conditions shows that the firm type with the less elastic demand is better off deviating to full disclosure and increasing its price.}
6 Discussion

A standard policy implication of the unraveling result in the persuasion game is that laws mandating disclosure of quality information are unnecessary. One obvious implication of our main result is that such a conclusion extends to all situations where pairwise monotonicity holds. There is however a critical difference between the welfare properties of our setting that allows for heterogeneous consumer types and the standard setting with only one type of consumer. In the latter case, information disclosure is clearly welfare improving, because it prevents the firm from selling a low quality product for which the consumer’s willingness to pay is below the production cost. With taste heterogeneity, it is no more clear that product information disclosure is desirable. It is straightforward to construct examples where non disclosure expands the set of firm and consumer types combinations for which the product is sold with a strictly positive social surplus. As the following example shows, it may be socially improving that no information is revealed even if the probability that the product is sold is unchanged from what it would be with full revelation.

\[ r = \begin{bmatrix}
    t_1 & t_2 & t_3 \\
    s_1 & 6 & 2.75 & 1.5 \\
    s_2 & 2 & 2 & 1.5
\end{bmatrix} \]

All type combinations have equal probability and costs are uniformly zero. In the fully revealing equilibrium, firm type \( s_1 \) sells at price 6 to consumer type \( t_1 \) alone and firm type \( s_2 \) sells to all consumer types at price 1.5. By contrast, if the firm has no means of certifying product information, it sells to consumer types \( t_1 \) and \( t_2 \) at price 2.375. Non disclosure improves social welfare by allowing consumer type \( t_2 \) to consume product type \( s_1 \) whereas full disclosure would instead have consumer type \( t_3 \) consuming product type \( s_2 \). Further note that the consumer also benefits remaining uninformed as he obtains a larger \( \text{ex ante} \) expected surplus. The above example could readily be modified to yield a situation where full
disclosure leads to the highest welfare and the sale probability is not affected by information disclosure. Hence it is clear that, if pairwise monotonicity holds, mandatory disclosure laws are unnecessary but there is no general argument suggesting that the provision of certified information by firms should be facilitated or prevented.

The remainder of the section discusses two extensions of the analysis.

More general communication. We have assumed that the consumer is not able to communicate information about his type to the firm before the pricing and disclosure stage. This is without loss of generality when our condition for the existence and uniqueness of a fully revealing equilibrium is satisfied; that is, the fully revealing equilibrium exists and remains unique under pairwise monotonicity even with two-way communication. But other interesting equilibria could be obtained when pairwise monotonicity is not satisfied, by adding communication from the consumer to the seller. To see this, consider the following match function with a uniform prior and no cost:

\[
\begin{array}{ccc}
 t_1 & t_2 & t_3 \\
 s_1 & 3 & 0 & 2 \\
 s_2 & 0 & 3 & 2
\end{array}
\]

There is a fully revealing equilibrium with profit $4/3$ and a non revealing equilibrium with profit $3/2$. This profit can be improved upon if the consumer can first reveal whether his type is $t_3$ or not (this can even be a cheap talk claim). If the consumer reveals that his type is $t_3$ the price offered by the seller is 2 and he discloses no information; otherwise the price is 3 and product information is revealed by the seller. This yields profit $5/3$ which is strictly higher than any equilibrium profit without information transmission from the consumer.\(^{18}\)

Competition. There is limited literature on product information disclosure and competition. The existing work such as Meurer and Stahl (1994), Board (2009) or Anderson and

\(^{18}\)We thank V. Bhaskar for suggesting this example.
Renault (2009), considers a two stage game where firms first disclose information and then choose prices. Results in these models are often in contradiction with the predictions of the monopoly setting. Board (2009) considers Mussa and Rosen preferences (Mussa and Rosen, 1978) where consumers are heterogeneous in their willingness to pay for quality, and finds that the firm that draws the lower quality in a duopoly may choose not to disclose it. Note that the match induced by such preferences is both statewise monotonic with respect to the firm’s type and with respect to the consumer’s type. In contrast, Meurer and Stahl (1994) and Anderson and Renault (2009) consider information that pertains to the horizontal match between the firm’s type and the consumer’s type and find that, in equilibrium, product information is revealed. Monopoly models in similar settings would predict that the equilibria that maximize the firm’s profit involve no disclosure or partial disclosure, as in the example based on match $r_D$ at the end of Section 4.

The source of the difference however is not so much competition per se, but rather the strategic effect of information revelation in the first stage on the competitor’s pricing in the second stage. In both cases, firms choose not to reveal information, or on the contrary, to reveal it, in order to avoid a Bertrand type situation in the second stage that would wipe out profits. If we think of product information disclosure and pricing as selected simultaneously, such a strategic effect would disappear, and the relevant condition might be a joint condition on the two match functions associated with the two competing firms. To see this in a simplified setting, assume that firm 2’s product type is commonly known and firm 2 knows firm 1’s type, $s$. Let the firm’s matches be given by the match function $r_1 = r_B$ on page 10 for firm 1, and $r_2 = 0$ when $t = t_1$ and $r_2 = 4$ when $t = t_2$ for firm 2. Consumer types are equally likely independently of firm 1’s type for which probabilities are $Pr(s_1) = 3/4$ and $Pr(s_2) = 1/4$. If prices are $p_1$ for firm 1 and $p_2$ for firm 2, the consumer buys from firm 1 if and only if $\Delta r(s, t) = r_1(s, t) - r_2(t) \geq p_1 - p_2$ (market is covered). The
following matrix describes $\Delta r$. In that case

$$
\Delta r = \begin{bmatrix}
  t_1 & t_2 \\
  s_1 & 1 & 2 \\
  s_2 & 2 & -1
\end{bmatrix}
$$

With zero costs, although match functions for each firm are pairwise monotonic, there is a pooling equilibrium with $p_1 = 5/4$ and $p_2 = 0$ where both consumer types buy from firm 1.

In the above example, the matrix describing $\Delta r$ is not pairwise monotonic. If it was, then no such pooling equilibrium could be sustained if firms play pure strategies in prices. In such an equilibrium a subset of product types $S' \subseteq S$ is not revealed to the consumer and the price $p_2$ of firm 2 is correctly anticipated by firm 1 and is constant across types in $S'$. Hence, given $p_2$, firm 1’s strategic problem is the same as under monopoly replacing $r(s, t)$ by $\Delta r(s, t) + p_2$ over $S'$. Following the same argument as in the first part of the proof of Theorem 1, and assuming again that the cost of firm 1 is not too high, the best response of firm 1 is necessarily fully revealing whenever $\Delta r$ is pairwise monotonic. This is for instance the case if firm 1’s match satisfies statewise monotonicity with respect to its own type, in which case $\Delta r$ is statewise monotonic with respect to firm 1’s type whatever the match function of firm 2. The uniqueness result in Theorem 1 may thus be generalized to a setting that allows for some competition.

We now illustrate with an example involving a continuum of consumer types where the fully revealing equilibrium can be shown to exist. There are two equally likely product types $s_1$ and $s_2$, zero cost, and the consumer’s type is uniformly distributed on $[0, 1]$. Let $\Delta r(s_1, t) = 1 + 5t$ and $\Delta r(s_2, t) = 2 + 3t$. In a pooling equilibrium the expected match difference for consumer type $t$ is $\Delta \bar{r}(\cdot, t) = 1.5 + 4t$, which is uniformly distributed on $[1.5, 5.5]$. Firm 1’s demand is $D_1(p_1, p_2) = \frac{1}{4} (5.5 - p_1 + p_2)$ and firm 2’s demand is $1 - D_1(p_1, p_2)$. Hence, candidate equilibrium prices are $p_1 = \frac{19}{6}$ and $p_2 = \frac{5}{6}$.

---

19That is, for every $s \in S$, there exists $t \in T$ such that $\Delta r(s, t) + p_2 > \gamma(s)$. 31
The marginal consumer $\bar{\ell}$ is such that $p_1 = \Delta r(\cdot, \bar{\ell}) + p_2$, i.e., $\bar{\ell} = 5/24$. If firm 1 of type $s_2$ discloses its information, then it could charge a higher price $\Delta r(s_2, \bar{\ell}) + p_2 = 63/24 + 20/24 = 83/24 > p_1$ without losing a single consumer type. Hence pooling cannot be an equilibrium.

By contrast, a fully revealing equilibrium can be constructed with prices $(p_{1,1}, p_{2,1}) = (\frac{11}{3}, \frac{4}{3})$ if firm 1 has type $s = s_1$ and $(p_{1,2}, p_{2,2}) = (\frac{5}{3}, \frac{1}{3})$ if firm 1’s type is $s = s_2$.

7 Concluding remarks

We considerably extends the well known unraveling argument according to which certified quality information is always revealed: we allow the relevant information to pertain to the horizontal match along with quality. We show that pairwise monotonicity is sufficient for existence and uniqueness of a fully revealing equilibrium outcome, and is also necessary in order for uniqueness to hold independent of the prior and costs. However, a fully revealing equilibrium always exists if the firm’s type and the consumer’s type are independent. We also find that pairwise monotonicity imposes no restriction on the shape of the full information demand curves for the different product types but rather, on the allocation of consumer types along the demand curves. We use this property to provide some intuition in terms of demand curves as to how pairwise monotonicity guarantees full disclosure of product information.

Our analysis provides new insights about the provision of certified product information by firms when consumer tastes are heterogenous. We highlight the role of two dimensions of heterogeneity. The first is reflected in a difference in the willingness to pay for a set of products that could potentially be sold by the firm. The second concerns horizontal match heterogeneity whereby different consumers rank products differently. Pairwise monotonicity is consistent with horizontal match heterogeneity only if consumer types may be ranked in terms of their willingness to pay for the product class under consideration. This is the case for instance for high quality video games in the example of the introduction. The same logic applies to other high quality high tech products. Think of top of the line cameras: expert
users are best matched with cameras that allow elaborate adjustments, whereas casual users prefer cameras that are easy to use, but we expect expert users to have a higher willingness to pay than casual users for a good camera, whatever its sophistication may be. These examples suggest that there is room for further research using representations of consumer taste heterogeneity in a multi dimensional product characteristics space. Such a framework would yield empirically relevant predictions as to when pairwise monotonicity might hold.\textsuperscript{20}

Future research should also explore how the analysis could be extended to consider disclosure of information by competing firms. At the end of Section 6 we hinted at a possible generalization of our results in a setting where only one competitor has private information. Multiple modeling and research questions remain. How can the analysis be extended when there is uncertainty about more than one firm’s product? Does each firm have the same information as consumers about its competitor’s product characteristics or do all firms share all product information? In the latter case, does a firm have the possibility to reveal information about its competitors’ products?\textsuperscript{21} What would be the market dynamics if firms get to learn about their competitor’s products or about demand over time?

\section*{A Appendix}

\textit{Non-existence of a fully revealing equilibrium due to correlated types.} Assume that costs are zero ($\gamma(s_1) = \gamma(s_2) = 0$) and consider the following match function and correlation matrix, 

\begin{footnotesize}
\footnote{One difficulty with an empirical application is that it requires observing consumer tastes for products that might never end up being marketed. Some relevant data might be found in market research undertaken prior to the introduction of a new product. Our analysis also suggests that market research could be used to determine the degree of informativeness of promotion campaigns in addition to eliciting the appeal of the products for different customer populations.}

\footnote{See Anderson and Renault (2009) for some results on the incentives to reveal pure match information on the competitor’s product.}
\end{footnotesize}
where we assume $0 < \rho < 2\varepsilon < 2$.22

\[
\begin{bmatrix}
  t_1 & t_2 \\
  s_1 & \rho & 2 \\
  s_2 & 2 & \rho \\
\end{bmatrix}
\quad \mu =
\begin{bmatrix}
  t_1 & t_2 \\
  s_1 & \frac{1-\varepsilon}{2} & \frac{\varepsilon}{2} \\
  s_2 & \frac{\varepsilon}{2} & \frac{1-\varepsilon}{2} \\
\end{bmatrix}
\]

If the firm fully reveals its type to the consumer, then it sets the price $p = 2$ and gets a profit equal to $2\varepsilon$ whatever its type. We show that if $\varepsilon$ is small enough, then the firm has an incentive to deviate from full revelation. We have to show that whatever the consumer’s belief after a deviation by the firm, $s_1$ or $s_2$ gets a profit which is strictly larger than $2\varepsilon$.

Notice that the consumer’s belief off the equilibrium path may depend on the observed price. This allows a large flexibility to punish the firm if it deviates from complete information disclosure. In this example, however, a deviation to the same price $p = 2\varepsilon + (1-\varepsilon)\rho$ will be profitable for at least one of the firm’s type whatever the consistent belief of the consumer. The idea is that this price is accepted by one type of the consumer with large probability $(1-\varepsilon)$ whenever a type $s_i$ of the firm makes the consumer believe that it is the other type $s_{-i}$, for $i = 1, 2$. Under some conditions on the game, the fact that two types want to imitate each others is sufficient to prevent full revelation of information (see, e.g., the “single crossing” property in Giovannoni and Seidmann, 2007), but is not sufficient in our framework as we also have to consider non-degenerated beliefs off the equilibrium path. For example, when $\rho = 0$, by setting the price to $2(1-\varepsilon)$, each type of the firm would be strictly better off when the consumer believes that it is the other type, but a fully revealing equilibrium can be constructed by setting the consumer’s belief off the equilibrium path to his prior belief.

We first observe that if $t_1$ or $t_2$ buys the good at price $p = 2\varepsilon + (1-\varepsilon)\rho$, then the expected

\[\text{22 Notice that this match is not pairwise monotonic. Otherwise, a fully revealing equilibrium would always exist by our theorem.}\]
profit of one of the two types of the firm is at least

$$\Pi = (1 - \varepsilon)(2\varepsilon + (1 - \varepsilon)\rho),$$

so the deviation would be profitable for one of those types whenever $\Pi > 2\varepsilon$, i.e., $\rho > \frac{2\varepsilon^2}{(1-\varepsilon)^2}$. This is possible under the assumption that $\rho < 2\varepsilon$ whenever $\varepsilon$ is small enough (take $\frac{\varepsilon}{(1-\varepsilon)^2} < 1$).

It remains to check that at least one of the consumer’s type $t_1$ or $t_2$ always accepts to buy at price $p$ off the equilibrium path. Let $m \in M(s_1) \cap M(s_2)$ be a message available to the firm whatever its type, and let $\mu_i$ be the consumer’s belief that the firm’s type is $s_1$ when the consumer’s type is $t_i$ and he observes the signal $(p, m)$ off the equilibrium path. The maximum price under which the consumer accepts to buy the good is $\bar{p}_1 = 2(1 - \mu_1) + \rho\mu_1$ when his type is $t_1$ and $\bar{p}_2 = 2\mu_2 + \rho(1 - \mu_2)$ when his type is $t_2$. The firm does not deviate from full revelation by sending $(p, m)$ only if $\bar{p}_1$ and $\bar{p}_2$ are both smaller than $p$, which yields $\mu_1 > 1 - \varepsilon$ and $\mu_2 < \varepsilon$. However this belief system is not consistent because it cannot be obtained by Bayes’ rule whatever the strategy of the firm.

**References**


